

Vortex–vortex interaction in two-component Bose–Einstein condensates

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The vortex–vortex interaction in two-component Bose–Einstein condensates is shown to present characteristic effects not possible in single-component condensates. In particular, vortices in different components undergo separate, but concentric, orbits for both same and opposite circulation. In addition, contrary to the well-known behavior in single-component condensates, vortices with the same circulation created in the less dense component merge in the presence of dissipation, allowing for the creation of metastable vortices with more than one quantum of circulation.

The experimental achievement of Bose–Einstein condensation (BEC) in trapped dilute alkali gases [1] has opened the possibility to explore fundamental properties of low temperature physics in a very controllable way. In particular, superfluid phenomena, well known from ⁴He physics [2], have been experimentally observed in BEC [3]. Among the phenomena related with superfluidity, the possibility of vortices with quantized circulation [4] has aroused a big interest. Due to the diluteness of the atomic condensates, accurate theoretical predictions are possible. In addition, in these systems, the diameter of the vortex core is typically three orders of magnitude larger than for superfluid Helium, allowing for a more straightforward optical observation. Numerous theoretical schemes have been suggested for generating vortices in BEC, see e.g. Refs. [5–11]. So far only two techniques have successfully generated vortices in condensates, namely the stirring with a blue detuned laser [12], and the coherent interconversion between the two components of a binary BEC [13]. In addition the creation of a vortex ring after the decay of a dark soliton has been recently reported [14]. The dynamics and stability of vortices in trapped Bose–Einstein condensates has recently also been a subject of active investigation [15].

In the recent years, the development of trapping techniques has allowed for the creation of multi-component condensates, which are formed by trapping atoms in different internal (electronic) states [16,17]. The multicomponent BEC, far from being a trivial extension of the single-component one, presents novel and fundamentally different scenarios for its ground state [18,19] and excitations [20]. In particular, it has been observed that the condensate can reach an equilibrium state characterized by the separation of the species in different domains [17]. The realization of vortices in binary condensates in recent experiments at JILA [13] has aroused the interest for the properties of vortices in multicomponent condensates [15,21,22]. Several interesting phenomena have been reported in this context, such as vortex ground states [23] and generation of persistent currents [22].

In this Letter we discuss the interaction of vortices in

a two-component BEC. The first part of the Letter is devoted to homogeneous miscible binary condensates. We consider in particular the case in which one vortex is created in each component. In single-component BEC (or classical gas) in absence of dissipation, two vortices with the same circulation orbit around each other, whereas vortices with opposite circulation move parallelly (vortex pair) [4]. However, for the case of asymmetric nonlinear interactions between the two components of a binary BEC, the velocity fields for the vortices are asymmetric. We show that in this scenario, vortices with the same circulation perform two concentric orbits. Interestingly, also vortices with opposite circulation perform concentric orbits, clearly differing from the one component behavior.

In the second part of this Letter we analyze the interaction of two vortices created in one of the components of an inhomogeneous binary BEC. In particular, we study the interaction of vortices with the same circulation, and show that the “buoyancy” effect [22] strongly distorts the vortex dynamics. In single-component BEC, vortices with the same circulation repel each other in the presence of dissipation, whereas the opposite can be true in a two-component BEC. This result strongly suggests that the creation of stable vortices with more than one quantum of circulation should be possible in a binary BEC.

Let us first consider the case of one vortex created in each of the components of a miscible binary BEC trapped in a 2D disc-shape box-like potential of radius R . The BEC can be considered homogeneous, except in a region of the size of the healing length, close to the box boundaries. The dynamics can be described in 2D if the mean-field interaction is smaller than the typical energy separation E_{\perp} in the third direction [25]. To generate the vortices we use the phase-imprinting (PI) technique [9], although other methods could be employed, as for example stirring methods [12], or the method employed in Ref. [13], which already involves two components.

For sufficiently low temperatures the dynamics is well described by two coupled Gross–Pitaevskii equations

$$i\hbar \frac{\partial}{\partial t} \psi_j(\mathbf{r}, t) = \left\{ -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + g_{jj} |\psi_j(\mathbf{r}, t)|^2 + \right.$$

$$g_{jl}|\psi_l(\mathbf{r}, t)|^2\} \psi_j(\mathbf{r}, t), \quad (1)$$

where $V(\mathbf{r})$ is the trap potential, $g_{jl} = 4\pi\hbar^2 a_{jl}/mL$ is the coupling constant (averaged over the frozen-out transversal direction) between the components j and l with the transversal length L , m is the atomic mass and a_{jl} the scattering length between j and l ($j, l = 1, 2$).

We have numerically simulated the creation of one vortex in each component by using PI. The creation of a vortex in one component modifies the density in the other one, due to the coupling in Eq. (1). For example, dark solitons in homogeneous two-component BEC can only be created at distances comparable to the healing length [26], $l_0 = \hbar/\sqrt{gnm}$, where n is the density of the sample. The latter does not apply to vortices, which can be created at distances significantly larger than l_0 , without being destroyed by the density fluctuations of the other component. Although we have numerically observed stable creation of vortices at distances larger than $20l_0$, typical distances of the order of $8l_0$ are used in the simulations below, since the time scale of the vortex-vortex dynamics is given by md^2/\hbar , where d is the distance between the vortices. Therefore shorter distances allow for faster dynamics. This facilitates the numerical simulations, and reduces instabilities during the vortex interaction. As already observed in recent experiments [13], the vortex core in one component is filled by the other component. Although the two components interact through the density, and therefore they are not directly experiencing the circulation of the other vortex, the filling of each core does feel the circulation created by the vortex in the same component, dragging in its way the vortex core in which it is placed. The result is an indirect interaction.

Let us postpone the discussion about the boundary effects and concentrate on the situation in which the radius R of the disc-shape trap is sufficiently large to neglect the boundary effects during the time scale of the simulations. In the following we consider $g_{11} = g_{22} = 1.05g_{12}$. These particular g_{ij} values have been chosen for simplicity, to guarantee a complete overlap between the wavefunctions of both components. In the case of ^{87}Rb , the coupling constants have the values $g_{11} : g_{22} : g_{12} \equiv 0.97 : 1.03 : 1.0$. These values also imply a large overlap, without separate domains, and therefore the dynamics we discuss below should be observable in ^{87}Rb .

In the symmetric situation, $n_{1,2} = n_0$, the qualitative behavior of single-component vortex-vortex interaction is recovered, namely, vortices with the same circulation rotate around each other, forming a vortex orbit, whereas vortices of opposite circulation move parallelly (vortex pair) through the fluid. The time evolution of the vortex cores is, however, different to the single-component one, since the effective masses of the vortex cores are different.

This picture breaks down when the $n_1 \neq n_2$. Fig. 1 shows the position of the vortex lines for the case of $n_1 = 2n_2 = 10^{11}\text{cm}^{-2}$, $d = 4\mu\text{m}$, and $R = 20d$, for

vortices with the same circulation. In this asymmetric scenario, contrary to the case of single-component vortices, vortices with the same circulation do not undergo an orbit, but two concentric ones. The inner orbit corresponds to the vortex in the denser component, whereas the outer one corresponds to the vortex created in the less dense one. Interestingly, such a behavior is also present for the case of vortices with opposite circulation.

In order to understand the formation of the planetary orbits when $n_1 \neq n_2$, let us consider the situation of a vortex line in the component 1 (2) at a position \vec{r}_1 (\vec{r}_2). The density of the component 1 (2) is considered constant and equal to n_1 (n_2) everywhere in the system, except in the core of the vortices, which will be modeled as a cylinder of radius l_1 (l_2). In the core of the vortex in the component 1 (2) the density of 1 (2) is considered zero, whereas the density of 2 (1) is considered $n_1 + n_2$, in such a way that the total density is flat. In the following calculation, we shall assume that $|\vec{r}_1|, |\vec{r}_2| \ll R$, i.e. the vortices are placed close to the trap center, and the vortex cores are far away from each other, $l_1, l_2 \ll |\vec{r}_2 - \vec{r}_1| = |\vec{r}|$. The conservation of linear momentum and energy in the system, up to order $\mathcal{O}((r_j/R)^2)$, $\mathcal{O}((l_j/r)^2)$, ($j = 1, 2$), lead respectively to the following equations:

$$n_1\vec{u}_1 + \kappa n_2\vec{u}_2 = \frac{n_2l_2^2 + \kappa n_1l_1^2}{r^2} \left(\vec{u} - 2 \left(\frac{\vec{r} \cdot \vec{u}}{r^2} \right) \vec{r} \right), \quad (2)$$

$$\left(\frac{r}{R} \right)^2 (n_1\vec{r}_1 \cdot \vec{u}_1 + n_2\vec{r}_2 \cdot \vec{u}_2) = -\frac{n_2l_2^2 + n_1l_1^2}{r^2} (\vec{r} \cdot \vec{u}), \quad (3)$$

where the velocity of the vortex in the j -th component is given by $\vec{v}_j = \vec{u}_j/r^2$, $\vec{u} = \vec{u}_2 - \vec{u}_1$, and $\kappa = 1$ (-1) for vortices with the same (opposite) circulation. Neglecting in Eq. (2) the relatively small rhs, and assuming R tending to ∞ , one obtains the solution $\vec{v}_2 \sim (\vec{r}_1 - \vec{r}_2)^\perp/|\vec{r}_1 - \vec{r}_2|^2$, and $\vec{v}_1 = -(\kappa n_2/n_1)\vec{v}_2$ [27]. The motion of the vortex lines is then determined by the first-order equations $d\vec{r}_j/dt = \vec{v}_j$. The trajectories provided by these equations (solid lines in Fig. 1) describe very well the numerically calculated vortex orbits. Note that since the velocity field is always tangential to the vector which joins the vortex lines, the distance between the vortices remains constant at any time, and consequently the angular frequency for both orbits is the same. Therefore, vortices of the same circulation are always at opposite angles of their respective orbits, whereas vortices with opposite circulation always occupy the same angle. Hence, the angular frequency for both orbits is the same. From this fact, it is easy to show that for vortices with the same/opposite circulation, initially placed at a distance d , the orbit of the outer vortex (say in component 1) has a radius $r_1 = dn_2/(n_1 \pm n_2)$, whereas the inner one has a radius $r_2 = dn_1/(n_1 \pm n_2)$. Note that for $n_1 = n_2$ the familiar one-component results are retrieved.

Let us discuss at this point the role of the boundaries on the dynamics. Our numerical analysis shows that the

finite size of the trap does not disturb the distance between the vortices, and, as in the case of $R \rightarrow \infty$, $\vec{r}_1 - \vec{r}_2$ moves in a circle. However, the coordinate $(\vec{r}_1 + \vec{r}_2)/2$, which for $R \rightarrow \infty$ moves in a circle as well, performs in the presence of a finite R a cycloid motion. As a result, the planetary orbits are not circular anymore, but will precess around a circular orbit, as one can observe in Fig. 2, where the same parameters of Fig. 1 are considered, except $R = 4.5d$. Nevertheless, similar to the case $R \rightarrow \infty$, the trajectory of the vortex in the denser component is always the inner trajectory.

In the last part of this Letter we analyze the case of a binary BEC trapped in a 2D harmonic potential of frequency ω . This resembles the situation in which vortices are currently created in ^{87}Rb in JILA [13]. In particular, we shall analyze the interaction of two vortices in the same component, which are created symmetrically from the condensate center, and with the same circulation. In homogeneous as well as single-component trapped condensates, and in the presence of dissipation, the vortices will spiral out from their initial orbit until they finally reach the boundaries of the condensate cloud, where they are destroyed [28]. We shall show that this is not the case in an inhomogeneous binary BEC.

In a recent paper [22], the dynamics of a filled vortex in a trapped binary BEC was analyzed. In an inhomogeneous potential, the lowest energy solution is reached when the component with smaller nonlinear self-interaction occupies the regions of highest density, i.e. the trap center. Therefore, in our case, if the less dense component is displaced from the trap center, it will tend to return to it. This effect is sometimes called the “buoyancy” effect [22]. This effect can counteract the tendency of an off-centered vortex to spiral out. In particular, it has been shown that for sufficiently large nonlinearity a vortex, created in the component with larger self-interaction at the center of the trap, can become metastable [22]. We employ a similar treatment to analyze the interaction of vortices. It consists in evaluating, for different distances between vortices, the free energy

$$E = \int d^3\mathbf{r} \left(\frac{\hbar^2}{2m} (|\nabla\psi_1(\mathbf{r})|^2 + |\nabla\psi_2(\mathbf{r})|^2) + V_1(\mathbf{r})|\psi_1(\mathbf{r})|^2 + V_2(\mathbf{r})|\psi_2(\mathbf{r})|^2 + \frac{g_{11}}{2}|\psi_1(\mathbf{r})|^4 + g_{12}|\psi_1(\mathbf{r})|^2|\psi_2(\mathbf{r})|^2 + \frac{g_{22}}{2}|\psi_2(\mathbf{r})|^4 \right). \quad (4)$$

In order to obtain a clear value of the free energy associated with every vortex configuration, instead of creating the vortices using the PI method discussed above, we generate them, as in Ref. [22], by evolving the Gross-Pitaevskii equation (1) in imaginary time, fixing at every iteration the phase of the component in which the vortices are created, in order to have the correct phase properties. Once imprinted in this way two vortices at position $(-d/2, 0)$ and $(d/2, 0)$, we calculate the energy

$E(d)$ following Eq. (4). For the case of single component condensates, one can show that $E(d)$ decreases with d , for any d . This reflects the fact that in the presence of dissipation the vortices tend to spiral out. For vortices created in the denser component (say 1), the situation is, however, completely different. Fig. 3 shows the case of ^{87}Rb ($a_{11} = 1.03a_{12}$, $a_{22} = 0.97a_{12}$, and $a_{12} = 5.7\text{nm}$), with a trap frequency $\omega = 2\pi \times 30\text{Hz}$ and $L = 1\mu\text{m}$, with number of particles $N_1 = 10^5$ and $N_2 = 8 \times 10^5$. One observes that $E(d)$ clearly grows with d for low d , until it reaches a maximal value. This maximum corresponds to the case in which the vortices are placed at the maxima of the unperturbed density distribution of the component 1 (see inset in Fig. 3). For larger distances the tendency of the vortices to spiral out is no more counteracted since the slope of the density distribution changes the sign. Therefore, vortices created sufficiently close to the center will tend to spiral inwards in the presence of dissipation, until eventually merging at the trap center. Once they merge, a vortex with double circulation will be created. Since due to its topology a vortex can only be destroyed if it reaches a region of zero density, the created vortex of double circulation will remain metastable at the trap center, where only inelastic processes will in practice limit its lifetime. Following similar arguments, the merging of n_v vortices should allow for the creation of a metastable vortex with n_v quanta of circulation, opening a new scenario which is naturally forbidden in both homogeneous single- and multi-component BEC, as well as in inhomogeneous single-component condensates.

Let us finally point out, that a similar argument applies for vortices with opposite circulation created in the denser component and moving parallelly in a toroidal trap [29]. In that case, the “buoyancy” effect counteracts the natural tendency of the vortices to mutually annihilate, allowing for the creation of a vortex pair, which will circulate around the torus center in a metastable orbit.

Summarizing, we have shown that the vortex-vortex interaction in multicomponent condensates, presents characteristic features such as planetary orbits for vortices placed in different components, and opens new interesting possibilities for creating metastable vortices with more than one quanta of circulation, as well as metastable vortex pairs in toroidal geometries.

The analysis of the vortex-vortex interaction has been limited to the case of 2D traps. Lower dimensional condensates are currently actively investigated, and very recently the experimental observation of 2D (and even 1D) condensates have been reported [24,25]. However, the results of the present Letter, open interesting questions concerning the behavior of vortices in 3D binary BEC. In particular, for linear vortices, if the relative density between the different components varies in the direction of the vortex lines, it should be expected that vortices created in different components will undergo at each transversal position planetary orbits of different periods

and radii. Consequently, non trivial distortions of the vortex lines are expected. In addition, the recent observation of vortex-rings [14] in condensates stimulates the investigation of the interaction of such more complicated structures in binary BEC. The analysis of these effects is beyond the scope of the present Letter, and will be the subject of further investigation.

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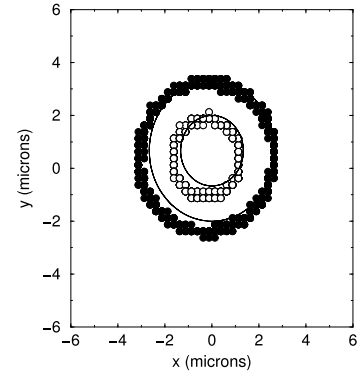


FIG. 1. Numerical (circles) and analytical (solid lines) results for the vortex orbits with $n_1 = 2n_2$, $d = 4\mu\text{m}$, $R = 20d$. The denser component occupies the inner orbit.

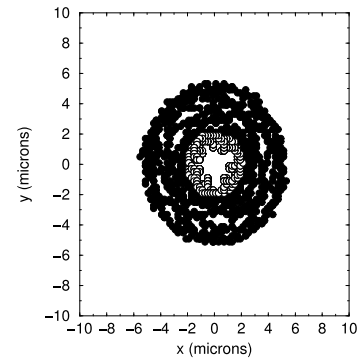


FIG. 2. Same as Fig. 1 but with $R = 4.5d$.

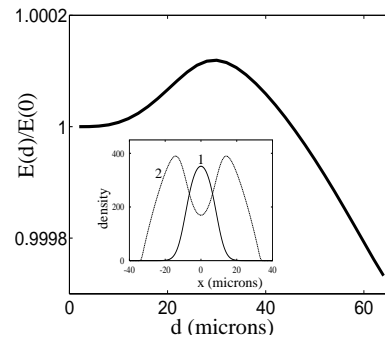


FIG. 3. Energy as a function of vortex distance (see text for details). Inset: Density distribution without vortices.